GENERAL EXPRESSION OF TEMPERATURE DEPENDENCE FOR HOLE AND ELECTRON CONCENTRATION IN MANY-VALLEY SEMICONDUCTORS

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Abstract – On the basis of charge neutrality equation, the expression for the temperature dependence of the charge carrier concentration in a non-degenerate many-valley semiconductor is derived.

Keywords: many-valley semiconductor; charge carrier concentration; charge neutrality equation

1. Introduction

In solid solutions when the starting compounds of such solutions are of qualitatively different band structure, with the change of compositions two or more valleys may be closely located according to their energy [1]. Such semiconductors behave either like two-valley ones or many-valley ones, accordingly. In the present work we derive a general expression for the temperature dependence of electron concentration in a many-valley semiconductor.

2. Derivation of general expression

We consider a many-valley $n$-type semiconductor which has $m$ valleys arranged in different points of the quasi-momentum $k$-space and located at $\delta E_i$ energy distance from a chosen zero level.
Let us consider a non-degenerate \( n \)-type compensated semiconductor with one type of singly ionized donors and acceptors. For simplicity, assume that the constant energy surfaces of each valley are of spherical symmetry in the vicinity of the minimum. It must be emphasized that the donor level corresponds to the valley from which the level is split off \([2, 3]\) and its energy level is calculated from the bottom of the corresponding valley.

Note that the general expression for the charge neutrality condition in case of the many-valley semiconductors can be written as

\[
\sum n_i + \sum n_{di} - N_d = \sum p_i + \sum n_{ai} - N_a ,
\]

where \( n_i \) is the concentration of free electrons of the \( i \)th valley, \( p_i \) is the concentration of free holes, \( n_{di} \) and \( n_{ai} \) are the concentrations of non-ionized donors and acceptors bounded with the \( i \)th valley, respectively, \( N_d \) and \( N_a \) are the concentrations of singly ionized donors and compensating acceptors, respectively.

If the band gap of the semiconductor is so large or the temperature is so low that in the considered temperature range the intrinsic conductivity is of no importance yet, and, taking into account the fact that in that case the Fermi level is located in the region near donor levels (at \( n \)-type conductivity) and, therefore, the concentration of free holes and non-ionized acceptors can be neglected, then the charge neutrality equation becomes

\[
\sum n_i + \sum n_{di} = N_d - N_a .
\]

If the counting of energy is performed with respect to the bottom of each \( i \)th valley, the energy distance from the Fermi level can be represented as \((-\delta E_i + E_F)\) (Fig. 1). Then the probability of occupation of the donor level related to the \( i \)th valley is expressed as follows:

\[
P(E) = \frac{1}{1 + \beta_i^{-1} \exp \left( \frac{E - E_F + \delta E_i}{kT} \right)} ,
\]

where \( E = -\delta_{di} \) and \( \beta_i \) is the number taking into account spin degeneracy of the \( i \)th level.
The concentration of the non-ionized donor levels is defined as

\[
n_{di} = \frac{N_d - \sum'_{j} n_{dj}}{1 + \beta_i^{-1} \exp \left( \frac{-e_d - E_F + \delta E_i}{kT} \right)},
\]

where \( \sum' n_{dj} \) is the concentration of the non-ionized donors associated with the rest of the valleys. The prime over the sum symbol denotes the term \( j = i \) is omitted.

Obviously,

\[
\sum_i n_{di} = \sum_j n_{dj} + n_{di}.
\]
Rearrangement of Eq. (4) gives

\[ n_{d_i} = \frac{N_{d_i} - \sum_i n_{d_i} + n_{d_i}}{1 + \beta_i^{-1} N_{c_i} \exp\left( \frac{\varepsilon_{d_i}}{kT} \right) + \frac{E_F - \delta E_i}{kT}}. \]  

(6)

Here, \( N_{c_i} \) is the effective density of states of the \( i^{th} \) valley and equals

\[ N_{c_i} = M_i \cdot 2 \left( \frac{2\pi m_i kT}{\hbar^2} \right)^{\frac{3}{2}}, \]  

(7)

where \( M_i \) is the number of equivalent valleys. Taking into account the expression

\[ n_i = N_{c_i} \exp\left( \frac{E_F - \delta E_i}{kT} \right), \]  

(8)

and denoting

\[ B_i = \beta_i^{-1} N_{c_i} \exp\left( \frac{\varepsilon_{d_i}}{kT} \right), \]  

(9)

we get a formula

\[ n_{d_i} = \left( \frac{N_{d_i} - \sum_i n_{d_i} + n_{d_i}}{n_i + B_i} \right) n_i, \]  

(10)

or taking into account Eq. (2), the latter can be written as

\[ n_{d_i} = \frac{n_i (n + N_{a})}{B_i} = \frac{n_i (n + N_{a})}{\beta_i^{-1} N_{c_i} \exp\left( \frac{\varepsilon_{d_i}}{kT} \right)}. \]  

(11)

This relationship makes it possible to calculate the concentration of non-ionized donor levels bounded to the \( i^{th} \) valley.

Substituting Eq. (11) into the charge neutrality equation (2) and taking into account that \( n = \sum n_i \),

we obtain

\[ n + (n + N_{a}) \sum_i \frac{n_i}{B_i} = N_{d_i} - N_{a}. \]  

(12)
or

\[
\frac{n(n + N_a)}{N_d - N_a - n} = \frac{1}{n} \sum \frac{n_i}{T_i B_i} .
\]  

(13)

Denoting the relative occupancy of the \(i^{th}\) valley by \(c_i = n_i/n\), one can obtain finally the following relation for Eq. (13):

\[
\frac{n(n + N_a)}{N_d - N_a - n} = \left[ \sum \frac{c_i}{p_i^{-1} N_i \exp\left(-\frac{\epsilon_i}{kT}\right)} \right]^{-1} .
\]  

(14)

This relation is defined as the general expression for the temperature dependence of the electron concentration in a many-valley semiconductor in a low-temperature region. From Eq. (14), at \(i = 1\), as a particular case, the well-known expression [4] follows:

\[
\frac{n(n + N_a)}{N_d - N_a - n} = p^{-1} N_c \exp\left(-\frac{\epsilon_d}{kT}\right) .
\]  

(15)

3. Conclusions

Thus, we have derived the general expression for the temperature dependence of the electron concentration in the non-degenerate many-valley \(n\)-type semiconductor at the low-temperature region. Using the above reasoning, one can obtain similar expressions for the case of many-valley \(p\)-type semiconductors:

\[
\frac{p(p + N_d)}{N_a - N_d - p} = \left[ \sum \frac{c_i}{p_i^{-1} N_i \exp\left(-\frac{\epsilon_i}{kT}\right)} \right]^{-1} .
\]  

(16)
References


