Armenian Journal of Physics, 2013, vol. 6, issue 1, pp. 7-34

THE COSMOLOGICAL REPULSION IN PRESENCE OF SCALARS FIELDS

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Received 12 November, 2012

Abstract–The recently discovered accelerated expansion of the Universe is of great interest in theoretical research on the evolution of the Universe. The cause of this expansion presumably is the presence of dark energy, which has been estimated to form up to 74% of the Universe and generates a “repulsive force”. In this paper a cosmological model is constructed which takes the dark energy into account in a Jordan–Brans–Dicke tensor-scalar theory with a non-minimally coupled scalar field in the presence of a cosmological scalar.

We consider 4 classes of cosmological models. The first class is the models of solutions in the proper frame of JBD theory with non-minimally coupled scalar field dominations, in the presence of cosmological scalar, which is becoming to the conventional cosmological constant Λ in Einstein frame under a conformal transformation. These models accept the possibility of uniform expansion and then of the accelerated expansion appear in cosmological times.

The second class considers cosmological models in the framework of the Jordan–Brans–Dicke (JBD) theory in the Einstein frame. The cases of scalar field and the presence of cosmological constant with the matter being described by barometric equation of state $P = \alpha\varepsilon$ (P is pressure, ε is energy density) are discussed separately. The analysis of obtained result is carried out in the light of modern observational data. It is shown that the contribution of the scalar field and Λ $(\Lambda > 0)$ in the case of $q = -1/2$ (q is “decelerating” parameter) compensate each other, thus leading to Einstein’s theory.

In the third class the problem is solved for the radiation dominated epoch. Numerical calculations are done for models with different values of the dimensionless coupling constant ζ.

In the last section a conformal coupled scalar field is considered.

Keywords: cosmology, cosmological scalar, dark energy, accelerating expansion

1. Introduction

Physically most reasonable and best treated version of tensor-scalar theory is the theory of Jordan–Brans–Dicke. Investigating transformational properties of metric in five-dimensional theory of Kaluza–Klein–Jordan, against supposition of Kaluza about constancy of g_{55}, noticed that the component is typical scalar. Let us consider this constant in the given time-space point proportional to the gravitational scalar, in other words, in the realization of Dirac’s hypothesis of variation of gravitational constant [1,2] suppose that g_{55} is replacing that constant. Jordan successfully formulated, different from GR, theory of gravitation [3]. (It would be worth to mention that nowadays there are data, about time variations of gravitational constant in order of 10^{-10} yrs^{-1} [4,5].) After some years, based on the Mach’s idea about influence of distant mass on the origin of inertia, Dicke and Brans [6] formulated analogical theory, physical prior of which does not coincide with Jordanian but field equations are similar. In JBD theory gravitational scalar $y(x) = g_{55}$ is generated with matter and non-gravitational fields, and precisely obeys to a wave type field equation, which has source in the form of trace of energy–momentum tensor of matter and non-gravitational fields. Influence of scalar field on the particle motion behaves not due to the immediate interaction but through caused by that field variation of metric tensor. At the same time, as in GR, field equations are reduced to the covariant divergence of energy-momentum tensor of matter and non-gravitational sources to zero, providing with that agreement with the demand of weak equivalence principle: probable neutral spinless particles and light rays move by geodesics.

In the work different presentations of JBD theory are considered, which arise after conformal transformation of metric. Mathematical equivalence establishing suggestions are formulated, which allow us by known precise solutions in the given presentation to generate new ones, particularly, by known solutions in JBD theory to get a new one in GR and opposite.

Some benefits of tensor-scalar theories are:

1. Superstring theories lead to the tensor-scalar theories of gravitation in the weak energy limit.
2. Taking into account of pre-friedmannian epoch of the Universe expansion is necessary for the agreement of cosmological evolution picture with the observational data. In GR transition from inflationary phase to the next phase is possible only after "fine tuning" of cosmological parameters, however the presence of gravitational scalar in tensor-scalar theory of gravitation allows us to implement phase transition smoothly, without any tuning.
3. Unlike the GR theory, tensor-scalar theories of gravitation foresee existence of gravitational radiation of monopole (from spherical) and dipole (from double sources) character, which will allows to test tensor-scalar theories of gravitation, it is related with the project of observatory on lasers interferometers for the detection of gravitational waves, and generated new wave of interests of investigation of non-stationary phenomena in different theories of gravitation.

2. Conformal Correspondence of the JBD and GR Theories

Let suppose that in the same many-fold two conformal-corresponding Reimann structures endowed $V4$ and $V'4$ spaces are give

$$d\,\overline{s}^{2} = ds^{2}\sigma^{2}\left(x\right) = \sigma^{2}\left(x\right) g_{\mu\nu} dx^{\mu} dx^{\nu}, \quad \overline{g}_{\mu\nu} = \sigma^{2}\left(x\right) g_{\mu\nu}, \quad g_{\mu\nu} = \sigma^{-2}\left(x\right)\overline{g}_{\mu\nu} \tag{1}$$

Besides mathematical content we try to give physical interpretation to the (1) conformal transformation by relating them with the scale transformation of units of measurements.

The idea of relationship between the using of different systems of units of physical quantities and the local conformal transformations goes back to Weil [7], Eddington [8], Dicke [9]. It is natural to consider that universal constants (speed of light and Plank constant) are invariant under conformal transformations:

$$\overline{c} = c, \quad \overline{\hbar} = \hbar . \tag{2}$$

We also assume that those 1-forms in the definition of which the metric tensor does not appear are unchanged under the transformations (1), for example $A_{\mu} = \overline{A}_{\mu}$ potential of an electromagnetic field. On the other hand, the components of the 4-velocity are being transformed according to

$$u^{\mu}=\frac{dx^{\mu}}{ds}=\frac{dx^{\mu}}{\sigma^{-1}d\,\overline{s}}=\sigma\overline{u}^{\mu},\;\; u_{\mu}=g_{\mu\nu}\cdot u^{\nu}=\sigma^{-1}\overline{u}_{\mu}$$

(a relation $u_{\mu}u^{\mu}=1$ in the space $\overline{V}_4$ conserves its form: $\overline{u}^{\mu}\overline{u}_{\mu}=1$, as it should be). It is easy to establish the relationship between physical quantities of different units on the base of (2). For example, for distance $\overline{l}=\sigma l$, for time $\overline{t}=\sigma t$, for mass $\overline{m}=\sigma^{-1}m$, for energy density $\overline{\varepsilon}=\sigma^{-4}\varepsilon$ and so forth. It is pertinent to note that if we postulate the conformal invariance of electrodynamics, then such a requirement would be conservation of the speed of light, the size of the elementary charge $e=\overline{e}$ and vector potential of electromagnetic field A_{μ} with respect to conformal transform.

Let us suppose that metric tensor of space abbeys to the equations of tensor-scalar theory of gravity, which are obtained as a result of independent variation of the action [10]

$$W=\int\sqrt{-g}\left[-F(\varphi)R+\frac{1}{2}\Phi(\phi)g^{\mu\nu}\phi_{\mu}\phi_{\nu}+L_m\right]dx^4 \tag{3}$$

with respect to $g_{\mu\nu}$ and ϕ. (Here L_m is density of Lagrange function of matter and non-gravitational fields, ϕ is gravitational scalar, and $(....)_{\alpha}=\partial(....)/\partial x^{\alpha}$). One can transform to $\overline{V}_4$ conformal corresponding space according to

$$\overline{g}_{\mu\nu}=\frac{F(\phi)}{F_0}g_{\mu\nu},\;\; F_0=\text{const.}$$

and for action (3) we have

$$\overline{W}=\int\sqrt{-\overline{g}}\left[-F_0\overline{R}+\frac{1}{2}\overline{g}^{\mu\nu}\psi_{\mu}\psi_{\nu}+\overline{L}_m\right]dx^4, \tag{4}$$

where

$$\psi_{\alpha}=\phi_{\alpha}\sqrt{3F_0\frac{F'^2}{F^2}+F_0\frac{\Phi}{F}},\quad F'=\frac{\partial F}{\partial\phi}.$$

Corresponding equations have the following form:

$$\overline{G}_{\alpha\beta}=\frac{1}{2F_0}\left(\overline{T}^{m}_{\alpha\beta}+\overline{T}^{s}_{\alpha\beta}\right),\quad \overline{g}^{\alpha\beta}\nabla_{\alpha}\psi_{\beta}=0,\quad \overline{T}^{s}_{\alpha\beta}=\psi_{\alpha}\psi_{\beta}-\frac{1}{2}\overline{g}_{\alpha\beta}\overline{g}^{\mu\nu}\psi_{\mu}\psi_{\nu}.$$

If $F_0=1/2k_0=c^3/16\pi G$ would be chosen then form of an action in $\overline{V}_4$ space corresponds with action minimally coupled ψ scalar field in GR, which satisfies homogenous wave equation, which lets us to formulate that Tensor-scalar Theory of Gravity (3) in a conformal corresponding space with metric tensor $\overline{g}_{\mu\nu}=\left(F(\phi)/F_0\right)g_{\mu\nu}$ is conformal equivalent to GR Theory with a source in the form of minimally coupled scalar field.

Suppose then that $g_{\mu\nu}(x)$ metric tensor in V_4 space obeys to the equations of JBD Theory

$$G^{\mu}_{\nu} = \frac{8\pi}{y} T^{\mu}_{\nu} + \frac{\nabla_{\nu} y^{\mu}}{y} + \zeta \frac{y_{\nu} y^{\mu}}{y^2} - \delta^{\mu}_{\nu} \left(\frac{\nabla_{\alpha} y^{\alpha}}{y} + \frac{\zeta}{2} \frac{y^{\alpha} y_{\alpha}}{y^2} \right), \tag{5}$$

$$\nabla_{\alpha} y^{\alpha} = \frac{8\pi T}{3+2\zeta}, \tag{6}$$

or in the equivalent form

$$R^{\mu}_{\nu} = \frac{8\pi}{y} \left[T^{\mu}_{\nu} - \delta^{\mu}_{\nu} \frac{1+\zeta}{3+2\zeta} T \right] + \frac{\nabla_{\nu} y^{\mu}}{y} + \zeta \frac{y_{\nu} y^{\mu}}{y^2},$$

$$R = -\frac{16\pi}{y} \frac{\zeta}{3+2\zeta} T + \zeta \frac{y_{\alpha} y^{\alpha}}{y^2}.$$

Here ζ is dimensionless coupling constant of JBD Theory, ∇_{α} notes covariant derivative with respect to x^{α}.

Post-Newtonian approximation of JBD theory leads to the necessity of normalization of gravitational scalar for to get asymptotes in the case of weak gravitational fields:

$$y(x) \to y_0 = \frac{2(2+\zeta)}{G(3+2\zeta)}. \tag{7}$$

Equations (5) and (6) are obtained by variation of the following action:

$$W = \int \sqrt{-g} \left[-\frac{y}{16\pi} \left(R - \zeta \frac{y_{\mu} y_{\nu}}{y^2} \right) + L_m \right] d^4 x \tag{8}$$

with respect to $g_{\mu\nu}$ and $y(x)$. Notice that action (8) is a particular case of (3), if set $F = y/16\pi$, ($F_0 = y_0/16\pi \equiv 1/2k$), $\Phi = \zeta\, y/8\pi$, $\phi_{\mu} = y_{\mu}/y$.

We transform into the conformal corresponding space $\tilde{V}_4$, the metric tensor of which is

$$\tilde{g}_{\mu\nu} = (y/y_0)^k g_{\mu\nu}, \tag{9}$$

while

$$\tilde{W} = \int \sqrt{-\tilde{g}} \left[-\frac{1}{2k} \left(\frac{y}{y_0} \right)^{1-k} \left(\tilde{R} - \frac{A}{2} \tilde{g}^{\alpha\beta} \frac{y_{\alpha} y_{\beta}}{y^2} \right) + \tilde{L}_m \right] d^4 x \ . \tag{10}$$

Here we introduced notations

$$A = (3+2\zeta) - 3(1-k)^2, \quad k = 1 - \sqrt{\frac{3+2\zeta - A}{3}}. \tag{11}$$

Variation of (10) with respect to $\tilde{g}_{\alpha\beta}$ and y leads to the equations

$$\tilde{G}^{\alpha}_{\beta} = k \left(\frac{y}{y_0} \right)^{k-1} \tilde{T}^{\alpha}_{\beta} + \left(\frac{A}{2} - k(1-k) \right) \frac{y_{\beta} y^{\tilde{\alpha}}}{y^2} + \delta^{\alpha}_{\beta} \left(k(1-k) - \frac{A}{4} \right) \frac{y_{\mu} y^{\tilde{\mu}}}{y^2} + \frac{1-k}{y} \left[\tilde{\nabla}_{\beta} y^{\tilde{\alpha}} - \delta^{\alpha}_{\beta} \tilde{\nabla}_{\mu} y^{\tilde{\mu}} \right], \tag{12}$$

$$\left(1-k\right)\tilde{R}-A\left(1+k\right)\frac{y_{\alpha}y^{\tilde{\alpha}}}{2y^{2}}+A\frac{\tilde{\nabla}_{\alpha}y^{\tilde{\alpha}}}{y}=0. \tag{13}$$

By combining trace of Eq.(12)

$$-\tilde{R}=k\left(\frac{y}{y_{0}}\right)^{k-1}\tilde{T}-\left(\frac{A}{2}-3k\left(1-k\right)\right)\frac{y_{\alpha}y^{\tilde{\alpha}}}{y^{2}}-3\left(1-k\right)\frac{\tilde{\nabla}_{\alpha}y^{\tilde{\alpha}}}{y} \tag{14}$$

with (13) we get equation defining gravitational scalar

$$\frac{\tilde{\nabla}_{\alpha}y^{\tilde{\alpha}}}{y}-k\frac{y^{\tilde{\alpha}}y_{\alpha}}{y^{2}}=k\left(1-k\right)\left(\frac{y}{y_{0}}\right)^{k-1}\frac{\tilde{T}}{3+2\zeta}. \tag{15}$$

So, in the result of (9) conformal transformations (5) and field equations (6) in the $\tilde{V}_{4}$ space transform into (12) and (15).

We redefine gravitational scalar as

$$\frac{\tilde{y}}{y_{0}}=\left(\frac{y}{y_{0}}\right)^{1-k}, \tag{16}$$

and introduce a new dimensionless constant

$$\tilde{\zeta}=\frac{A}{2\left(1-k\right)^{2}}=-\frac{3}{2}+\frac{3+2\zeta}{2\left(1-k\right)^{2}}. \tag{17}$$

In the new notations action (8) takes a form

$$\tilde{W}=\int\sqrt{-g}\left[-\frac{\tilde{y}}{16\pi}\left(\tilde{R}-\tilde{\zeta}\frac{\tilde{y}_{\mu}\tilde{y}_{\nu}}{\tilde{y}^{2}}\right)+\left(\frac{\tilde{y}}{y_{0}}\right)^{1-k^{2}}L_{m}\right]d^{4}x. \tag{18}$$

Equations of JBD theory are invariant under conformal transformations for any $k\neq1$ if we redefine gravitational constant and denoted coupling constant has a form, according to (16) and (17) (in the case of $\tilde{\zeta}=\zeta,\ k=2$).

3. Einstein frame of JBD theory [10]

Lets consider action (10) for the case, when the power degree of conformal transformation (9) $k=1$ $(\mathrm{A}=3+2\zeta)$. We introduce now denotation

$$\phi_{\alpha}=\frac{y_{\alpha}}{y}\sqrt{\frac{\left(3+2\zeta\right)y_{0}}{16\pi}} \tag{19}$$

and rewrite action (10) in the form of

$$\tilde{W}=\int\sqrt{-\tilde{g}}\left[-\frac{\tilde{R}}{2k}+\frac{1}{2}\tilde{g}^{\alpha\beta}\varphi_{\alpha}\varphi_{\beta}+\tilde{L}_{m}\right]d^{4}x, \tag{20}$$

where, as before, we have

$$2k = \frac{16\pi}{y_0} = \frac{8\pi G(3+2\zeta)}{2+\zeta}. \tag{21}$$

Considering case is a particular case of (10) and action (20) formally coincides with the form of Einstein–Hilbert action with minimally coupled scalar field. Action (20) corresponds to the equations

$$\tilde{G}_{\alpha\beta} = k\left(\tilde{T}^{m}_{\alpha\beta} + \phi_\alpha\phi_\beta - \frac{1}{2}g_{\alpha\beta}\tilde{g}^{\mu\nu}\phi_\mu\phi_\nu\right), \tag{22}$$

$$\tilde{g}^{\alpha\beta}\tilde{\nabla}_\alpha\phi_\beta = 0. \tag{23}$$

Note that

- equation (23) is a consequence of covariant invariance of $\tilde{G}_{\alpha\beta}$,
- Einstein constant of gravity is renormalized according to (21)

So, one can proved that equations of JBD theory in the result of conformal transformation (9) with $k=1$ becomes to the equations of GR with denotated Einstein constant of gravity and with the source in the form of sum of energy-momentum tensor of matter, non-gravitational fields and minimally coupled scalar field.

In the other words, conformal transformations (9) transform JBD theory from proper frame to the Einstein frame. Whereas, if in the proper frame gravitational scalar changes from point to point proportional to the Newton constant *G*, but universal constants *c*, $\hbar$ and particle mass remain unchanged, then in the Einstein frame *G*, *c* and $\hbar$ are constant, but particle mass changes in different points, $\tilde{m} = (y/y_0)^{-1/2}\, m$.

We transform $\breve{V}_4$ into the different space by using Bekeinstein results [11], conformal correspondence of which with the initial space V_4 is established according to

$$\breve{g}_{\mu\nu} = \frac{1}{4}z^{(n+1)/n}\left[1+z^{-n}\right]^2 g_{\mu\nu}, \tag{24}$$

$$\psi = \frac{6}{\tilde{k}}\left(\frac{z^n-1}{z^n+1}\right), \quad n = \sqrt{\frac{3+2\zeta}{3}}, \quad z = \left(\frac{y}{y_0}\right)^n. \tag{25}$$

In this case (8) is transformed into

$$\breve{W} = \int\sqrt{-\breve{g}}\left[-\frac{1}{2k}\breve{R}\left(1-\frac{1}{6}k\psi^2\right) + \frac{1}{2}\breve{g}^{\alpha\beta}\psi_\alpha\psi_\beta + \breve{L}_m\right]d^4x. \tag{26}$$

which proves that equations of JBD theory are transformed into equations of GR theory with the source in the form of non-gravitational fields and conformal-coupled massless scalar field ψ, by means of conformal transformations (24), (25), satisfying the equation of Penrose–Cherenkov–Tagirov

$$\breve{g}^{\alpha\beta}\breve{\nabla}_{\alpha}\psi_{\beta}-\frac{1}{6}\breve{R}\psi=0. \tag{27}$$

We are skipping the member $-(1/2k)\breve{R}$ in the action (26) and adding Higgs potential $(\lambda\psi^4/12)$. Then conform transforming gotten in the result behavior

$$\breve{W}=\int\sqrt{-\breve{g}}\left[-\frac{1}{12}\breve{R}\psi^2+\frac{1}{2}\breve{g}^{\alpha\beta}\psi_{\alpha}\psi_{\beta}-\frac{1}{12}\lambda\psi^4+\breve{L}_m\right]d^4x \tag{28}$$

by taking into account conformal factor, so in the result potential of scalar field becomes into the constant and so that "kinetic" term is compensated by additions contracted with a conformal factor.

This transformations satisfy to the above mentioned conditions (analogically it is considered in [10])

$$\hat{\psi}=\psi/\chi=\varepsilon=\text{const},\quad \psi=\varepsilon\chi,\quad \breve{g}_{\alpha\beta}=\chi^2\breve{g}^{\alpha\beta}. \tag{29}$$

Then we introduce

$$\hat{k}=-6/\varepsilon^2 \quad \text{and} \quad \Lambda=(1/2)\lambda\varepsilon^2,$$

So that

$$\hat{W}=\int\sqrt{-\hat{g}}\left[-\frac{1}{2\hat{k}}\left(\hat{R}+2\Lambda\right)+\hat{L}_m\right]d^4x. \tag{30}$$

This result can be critically interpreted so: violation of conformal symmetry can lead to the induction of gravitational field with conformal coupled scalar field.

The recently discovered accelerated expansion of the Universe is of current interest in theoretical research on the evolution of the Universe. The cause of this behavior is presumably the presence of dark energy, which has been estimated to form up to 74% of the Universe and generates a "repulsive force". In this paper a cosmological model is constructed which takes the dark energy into account in the Jordan–Brans–Dicke tensor-scalar model with a non-minimally coupled scalar field in the presence of a cosmological scalar.

In this review we consider 4 classes of cosmological models.

In the first class are the models of solutions in the proper frame of JBD theory with non-minimally coupled scalar field dominations, in the presence of cosmological scalar, while the cosmological scalar is becoming to the conditional cosmological constant in Einstein frame in the case of a conformal transformation. These models accept the possibility of uniform expansion and then of the accelerated expansion appear in cosmological times.

The second class considers cosmological models in the framework of the Jordan–Brans–Dicke (JBD) theory in the Einstein frame. The cases of scalar field and the presence of cosmological constant with the matter being described by barometric equation of state $P=\alpha\varepsilon$ (P is pressure, ε is energy density) are separately discussed. The analysis of obtained results is carried out in the light of modern observational data. It is shown that the contribution of the scalar field and Λ $(\Lambda > 0)$ in the case of $q = -1/2$ (q is "decelerating" parameter) compensate each other, thus leading to Einstein's theory.

In the third one, the problem is solved for the radiation dominated epoch. Numerical calculations are done for models with different values of the dimensionless coupling constant ζ.

In IV class is considered the tensor-scalar variant of the theory of gravitation with the conform-connected scalar field is considered. Various cosmological models are considered, the possibility of evolutionary development with the accelerated expansion of the Universe is discussed.

I. Cosmological Scalar in the Jordan-Brans-Dicke Theory

In this section, the cosmological problem in the proper representation of the JBD theory is considered in presence of non-minimally coupled scalar field. As it will be shown, introduction of the cosmological scalar provides a possibility for the transition from the decelerated expansion of the Universe to the accelerated one. It was noted in [12-17] that the modern conceptions of the Universe give rise to the introduction of the cosmological constant in the GR, therefore it is worth introducing a similar quantity in the JBD theory. Having assumed that the field corresponding to this quantity should be scalar but cannot be dynamical (its changes should be controlled by the gravitational scalar $y = y\left(x^{\mu}\right)$), we introduce the cosmological scalar $\varphi = \varphi\left(y\right)$ in the JBD theory action similar to the introduction of the cosmological constant in the GR action [12]:

$$W = \frac{1}{c}\int\sqrt{-g}\left[-\frac{c^4}{16\pi}y\left(R + 2\varphi\left(y\right) - \varsigma\frac{y^{\mu}y_{\mu}}{y^2}\right) + L_m\right]d^4x. \tag{I.1}$$

Here, ς is a dimensionless coupling constant of the JBD theory. The presence of the cosmological scalar means that, in addition to the kinetic term, the potential one is also considered for the scalar field.

Equating to zero the independent variations of the action (I.1) with respect to $g^{\alpha\beta}$ and y, we obtain the JBD theory equations with the cosmological scalar

$$\nabla_{\alpha}y^{\alpha} = \frac{k\overset{m}{T}}{3+2\zeta} + \frac{2y}{3+2\zeta}\left(\varphi - y\frac{\partial\varphi}{\partial y}\right),\; k = \frac{8\pi}{c^4}, \tag{I.2}$$

$$\begin{aligned} G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}R = \frac{k}{y}\left(\overset{m}{T}{}^{\mu}_{\nu} - \delta^{\mu}_{\nu}\frac{\overset{m}{T}}{3+2\zeta}\right) + \frac{\nabla_{\nu}y^{\mu}}{y} + \\ +\zeta\left(\frac{y_{\nu}y^{\mu}}{y^2} - \frac{1}{2}\delta^{\mu}_{\nu}\frac{y_{\alpha}y^{\alpha}}{y^2}\right) + \frac{\delta^{\mu}_{\nu}}{3+2\zeta}\left[\left(1+2\zeta\right)\varphi + 2y\frac{\partial\varphi}{\partial y}\right] = k\left(\frac{\overset{m}{T}{}^{\mu}_{\nu}}{y} + \overset{c}{T}{}^{\mu}_{\nu}\right), \end{aligned} \tag{I.3}$$

where

$$\overset{m}{T}{}_\nu^\mu = -P_m \delta_\nu^\mu + (\rho_m + P_m) U_\nu U^\mu,$$

$$\overset{c}{T}{}_\nu^\mu = \frac{1}{k}\left[\frac{1}{y}\left(\nabla_\nu y^\mu - \delta_\nu^\mu \nabla_\alpha y^\alpha\right) + \frac{\zeta}{y^2}\left(y_\nu y^\mu - \frac{1}{2}\delta_\nu^\mu y_\alpha y^\alpha\right) + \delta_\nu^\mu y \varphi(y)\right]. \qquad \text{(I.4)}$$

Here $\overset{m}{T}_{\mu\nu} = \left(2/\sqrt{-g}\right)\delta\left(\sqrt{-g}L_m\right)/\delta g^{\mu\nu}$ is the energy-momentum tensor of the matter, and $\overset{c}{T}{}_\nu^\mu$ is caused by the presence of the scalar field y, P_m and ρ_m are the pressure and the density of the matter (dust and radiation), considered as a perfect fluid, $U^\alpha = dx^\alpha / d\tau$ is 4-dimensional velocity satisfying the condition $U_\alpha U^\alpha = 1$. Conceivably, long-range massless scalar field with potential $y = y(x^\mu)$ (gravitational scalar) replaces the Newtonian gravitational constant G at each point, and according to Mach's ideas [18] it is a background created by the whole energy in the Universe. An essential feature of the JBD theory scalar field, which distinguishes it from the fields of other tensor-scalar theories (Kaluza-Klein theory, string theory), is that it does not interact directly with matter. This circumstance gives rise to consider the JBD scalar field in itself without matter and radiation, which could probably allow revealing its role in the formation of dark matter and dark energy.

In connection with the aforesaid, it is of interest to remind how the JBD theory was constructed. The idea of the existence of scalar field has very naturally arisen in the attempt to unify the gravitation and electromagnetism. It was found out that the five-dimensional formulation of the Einstein gravitation and Maxwell theory should be much easier than the four-dimensional one. The conclusion was made after the extended group-theoretical analysis according to which the property of the invariance of the Einstein-Maxwell theory in 5D is much more symmetric. It was noticed that the unified group of arbitrary four-dimensional coordinate transformations and gauge transformations of the electromagnetic field potential is isomorphic to the coordinate transformations group of five-dimensional Riemannian space. With respect to this the field equation of the projective theory [19-21] as well as modifications of the unified theory proposed by Pauli [22] are invariant. In this isomorphic group X^μ are transformed as 5-vectors, which allows to construct an additional invariant $y = g_{\mu\nu}X^\mu X^\nu$. To ensure that the GR equations reduced to 4D are equivalent to the Einstein-Maxwell field equations system, the requirement of constancy for the scalar $y = 1$ is necessary. So, directly and without speculations or additional hypotheses, as a result of physical analysis, an idea of the generalization of the gravitation theory arose, in which (in addition to the tensor and vector fields) there is a scalar field with the potential y. One of physically substantial and fully developed versions is the JBD theory, in which the gravitational scalar does not interact directly with the matter. Its existence is only manifested by the impact on the particle's motion.

Now we turn to the investigation of the cosmological model in JBD theory caused by the presence of non-minimally coupled scalar field. It makes sense to omit the possible contributions from all kinds of matter and determine the role of the scalar field, non-minimally coupled with the gravitation. We use spatially-flat Friedmann–Robertson–Walker metrics, assuming the space-time homogeneity and isotropy [19]:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \quad y = y(t), \tag{I.5}$$

where $k = 0$ stands for the spatially-flat case, $R(t)$ is the scale factor. In the case of the flat model, the cosmological equations in the Jordan proper frame take the following form:

$$\frac{\ddot{y}}{y} + \frac{3\dot{y}\dot{R}}{yR} = \frac{2}{3+2\zeta}\left(\varphi - y\frac{\partial\varphi}{\partial y}\right), \tag{I.6}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = -\frac{\ddot{y}}{y} - \frac{2\dot{R}\dot{y}}{Ry} - \frac{\zeta}{2}\frac{\dot{y}^2}{y^2} + \varphi, \tag{I.7}$$

$$3\frac{\dot{R}^2}{R^2} = \frac{1}{2}\zeta\frac{\dot{y}^2}{y^2} - 3\frac{\dot{R}\dot{y}}{Ry} + \varphi, \tag{I.8}$$

where the dot denotes the time derivative. If we determine φ from (I.8) and substitute it in (I.6) and (I.7), then (I.6) will exactly coincide with (I.7), that is for any choice of φ we have two independent equations.

The choice of $\varphi = \Lambda y/y_0$, where y_0 is the modern value of y and Λ is the Einstein theory cosmological constant, is caused by the fact that in this case under the conformal transformation $g'_{ik} = yg_{ik}$ we obtain the Einstein theory with cosmological constant (Einstein frame of the JBD theory). With this form of the function $\varphi(y)$, the integration of the subtract of equations (I.7), (I.8),

$$2\frac{\ddot{R}}{R} - 2\left(\frac{\dot{R}}{R}\right)^2 + \frac{\ddot{y}}{y} - \frac{\dot{y}^2}{y} + (\zeta+1)\frac{\dot{y}^2}{y^2} - \frac{\dot{R}}{R}\frac{\dot{y}}{y} = 0, \tag{I.9}$$

by taking into account the equation (I.6),

$$\frac{\ddot{y}}{y} = -3\frac{\dot{R}}{R}\frac{\dot{y}}{y}, \tag{I.10}$$

allows one to obtain the following result:

$$\frac{\dot{y}}{y} = c_- y^{(\sigma+1)/2} + c_+ y^{(\sigma-1)/2}, \quad \sigma = \sqrt{3(3+2\zeta)}\,. \tag{I.11}$$

After the substitution of (I.11) into (I.8) and taking into account (I.10), from the equation

$$\frac{1}{3}\left(\frac{\ddot{y}}{y}\right)^2-\frac{\zeta}{2}\left(\frac{\dot{y}}{y}\right)^2-\frac{\ddot{y}}{y}-\Lambda\frac{y}{y_0}=0$$

we obtain the following relation between the constants:

$$c_-c_+=-\Lambda/(3+2\zeta)\,y_0\,, \tag{I.12}$$

which allows to conclude that for $\Lambda=0$ in (I.11) either c_- or c_+ is zero, and for $\Lambda\neq 0$ the constants c_- and c_+ have opposite signs if $\Lambda>0$ and $\zeta>-3/2$. We consider two separate cases.

(a) First we assume that $\Lambda=0$. From equation (I.8) we obtain the following relation between the functions $\dot{R}/R$ and $\dot{y}/y$:

$$\frac{\dot{R}}{R}=-\frac{(3\pm\sigma)}{6}\frac{\dot{y}}{y}. \tag{I.13}$$

From (I.10) it follows that the plus sign corresponds to $\dot{y}=-|\,c_-\,|\,y^{(\sigma+3)/2}$, and the minus sign corresponds to the case $\dot{y}=|\,c_+|\,y^{(3-\sigma)/2}$.

As a result, for the scale factor we have

$$R_\pm=R_0\left(\frac{y_0}{y_\pm}\right)^{\pm(\sigma\pm3)/6}, \quad \left(\frac{y_0}{y_\pm}\right)^{\pm1}=\left[1+|\,c_\mp|\left(\frac{\sigma\pm1}{2}\right)y_0^{\pm(\sigma\pm1)/2}(t-t_0)\right]^{2/(\sigma\pm1)}, \tag{I.14}$$

where $t=t_0$, $R=R_0$, $y=y_0$ correspond to the present moment.

It is convenient to present the deceleration parameter $q=-R\ddot{R}/\dot{R}^2$ in the following form:

$$q=-\frac{\ddot{R}}{\dot{R}^2}R=3-\frac{R''R}{R'^2} \tag{I.15}$$

where the prime means the derivative with respect to y. Then, from (I.14) we have

$$q_\pm=\frac{2\sigma}{\sigma\pm3}. \tag{I.16}$$

Based on (I.4), it is possible to introduce by analogy the concepts of the pressure and the energy density of the scalar field, $T^\mu_{y\nu}=-P_y\delta^\mu_\nu+(\rho_y+P_y)\,u_\nu u^\mu$, in accordance with the relations

$$\rho_y^\mp=\frac{1}{2k}(\zeta+\sigma\pm3)\,c_\mp^2y^{(3\pm\sigma)}, \tag{I.17}$$

$$P_y^\mp=\frac{1}{2k}\left(\zeta+\frac{\sigma\pm3}{3}\right)c_\mp^2y^{(3\pm\sigma)}, \tag{I.18}$$

from which we find $1/3<P_y^-/\rho_y^-<1$, $1<P_y^+/\rho_y^+<\infty$ at $0<\zeta<\infty$ and, accordingly, the effective velocity of the elementary excitations (sound) arising in such medium can be defined as [12]

$$v^2=\frac{dP_y^\pm}{d\rho_y^\pm}=1\mp\frac{2(\sigma\pm3)-4}{3\zeta\pm2(\sigma\pm3)}. \tag{I.19}$$

It follows from (I.16) that $q = 0$ is possible only in the case $\zeta = -3/2$, however this value of the JBD theory dimensionless parameter is initially excluded from the consideration at formulation of the theory [19], because in this special case as a result of the variation of the action ten equations are obtained for 11 unknown quantities. It is interesting to note that for $\zeta = -3/2$ one has $P_y = -\rho_y/3$ and $c^2 < 0$. Negative pressure can be reasonably interpreted, however, the imaginary sound speed in liquid is rather doubtful. Thus, under the condition $\zeta > -3/2$ and in the presence of only scalar field in JBD theory, we conclude that q_+ can only accept positive values, which corresponds to the deceleration of the expansion of the Universe, and q_- is positive for $\zeta > 0$ and can be negative for $-3/2 < \zeta < 0$; however $\lim_{\zeta \to \pm 0} q_- = \pm\infty$, which confirms once again the advisability of the assumption that ζ is positive.

b) Now we consider the case $\Lambda \neq 0$. As it was already noted, in the expression for $\dot{y}/y$ from (I.11) in the case of $\Lambda > 0$, the constants c_- and c_+ are distinct from zero and they have different signs. Their signs coincide in the case $\Lambda < 0$.

From (I.10) and (I.11) we have

$$\frac{R'}{R} = -\frac{1}{6y} \frac{c_-(\sigma+3) y^{\sigma/2} + c_+(3-\sigma) y^{-\sigma/2}}{c_- y^{\sigma/2} + c_+ y^{-\sigma/2}}. \tag{I.20}$$

Taking into account (I.15), we find

$$q = 2 - \frac{6}{\left[c_-(\sigma+3) y^{\sigma/2} + c_+(3-\sigma) y^{-\sigma/2}\right]^2} \left[c_-^2(\sigma+3) y^{\sigma} + c_+^2 (3-\sigma) y^{-\sigma} + c_- c_+ \left(6 - 2\sigma^2\right)\right] \tag{I.21}$$

If one of the constants of integration (c_- or c_+) is zero, the value of (I.21) coincides with that for (I.16). In the presence of Λ, in addition of positive values for q, it is possible to obtain $q = 0$ and $q < 0$.

The zero acceleration of the expanding Universe corresponds to a certain value of y_1 satisfying the equation

$$c_-^2(\sigma+3) y_1^{2\sigma} + 4\sigma c_- c_+ y_1^{\sigma} + c_+^2(\sigma-3) = 0,$$

from which one has

$$y_1^{\sigma} = \frac{c_+}{c_-} \frac{-2\sigma \pm \sqrt{3\sigma^2+9}}{\sigma+3} = \frac{2\Lambda\left(\sqrt{1+2\zeta/3} \mp \sqrt{1+\zeta/2}\right)}{y_0 c_-^2 \left(\sqrt{1+2\zeta/3}+1\right)(3+2\zeta)}. \tag{I.22}$$

It follows from the expansion of the Universe that

$$R = c\dot{y}^{-1/3} \to \infty, \quad \dot{y} \to 0 \Rightarrow \ddot{y} < 0,$$

and from $\dot{y}=0$, taking into account that the signs of c_- and c_+ are different, we obtain the limiting value for y:

$$y_n = \left| c_+ / c_- \right|^{1/\sigma} . \tag{I.23}$$

It is easy to see from the expression

$$\ddot{y} = c_- \frac{3+\sigma}{2} y^{(1+\sigma)/2} - c_+ \frac{\sigma-3}{2} y^{(1-\sigma)/2} ,$$

that $\ddot{y}<0$ for $c_-<0$ and $c_+>0$, in this case $\dot{y} = |\,c_-\,| y^{(3+\sigma)/2} \left(|y_n/y\,|^{\sigma} -1 \right)$.

For $y(t_0)>y_n$ one has $\dot{y}<0$, and y decreases asymptotically tending to y_n. However if $y(t_0)<y_n$, then $\dot{y}>0$ and y asymptotically increases to y_n. In both cases at $t\to\infty$ one has $R\to\infty$, $y = y_n + Ae^{-(t-t_0)/\tau}$, where $\tau = 2 \big/ \sigma \sqrt{|\,c_- c_+\,| y_0} c$ is the characteristic time, c is the light velocity, and the sign of the constant of integration A depends on the sign of $\dot{y}$.

As regards the deceleration parameter q, from (I.22) it can be presented in the following form:

$$\frac{\ddot{R}R}{\dot{R}^2} = 1 + \frac{3\dot{y}\left[|\,c_-\,| (\sigma+3)(\sigma+1) y^{\sigma/2} - |\,c_+\,| (\sigma-3)(\sigma-1) y^{-\sigma/2} \right]}{y^{3/2} \left[|\,c_-\,| (\sigma+3) y^{\sigma/2} + |\,c_+| (\sigma-3) y^{-\sigma/2} \right]^2} ,$$

from which it follows that in the limit $\dot{y}\to 0,$ one has $\ddot{R}\to 1$, which corresponds to the accelerated expansion of the Universe. Thus, the scalar field value

$$y_1 = y_n \left(\frac{2\sigma \pm \sqrt{3\sigma^2+9}}{\sigma+3} \right)^{1/\sigma}$$

at the moment of the zero acceleration can be either larger or smaller than the limiting value y_n.

The system of equations (I.10), (I.11) can be explicitly solved. The corresponding solution is presented in the permanent form [23]

$$\begin{gathered} \frac{t}{t_1} = u^{(\sigma+1)/2\sigma/} {}_2F_1\left(1, \frac{\sigma+1}{2\sigma}, 1+\frac{\sigma+1}{2\sigma}, u\right), \\ \frac{R}{R_1} = \left[\frac{u^{(\sigma+3)/2\sigma}}{1-u} \right]^{1/3} , \quad \frac{y}{y_n} = u^{-1/\sigma}, \end{gathered} \tag{I.24}$$

with the parameter $0\le u\le 1$. In this formula ${}_2F_1(a,b;c;x)$ is the hypergeometric function and we have used the notation $t_1 = 2\left|c_+/c_-\right|^{(\sigma-1)/2\sigma} \big/ c_+(\sigma+1)$. In the limit $u\to 0$ one has $t/t_1 \approx u^{(\sigma+1)/2\sigma}$ and we have the following asymptotic behavior

$$y \propto t^{-2/(\sigma+1)}, \;\; R \propto t^{(\sigma+3)/3(\sigma+1)}, \;\; q \approx 2\sigma/(\sigma+3), \;\; t\to 0,$$

for the scalar field, scale factor and the deceleration parameter. This limit corresponds to early stages of the cosmological expansion. In the limit $u \to 1$ one has

$$t \approx -t_2 \ln(1-u), \quad t_2 = \left|c_+/c_-\right|^{(\sigma-1)/2\sigma} \Big/ c_+\sigma,$$

and the asymptotics for the scalar field and scale factor have the form

$$y \approx y_n\left[1+(1/\sigma)e^{-t/t_2}\right], \quad R \approx R_1 e^{t/3t_2}, \quad q \approx -1, \quad t \to \infty .$$

This limit corresponds to late stage of the cosmological expansion. Hence, at late stages the scalar field tends to the constant value y_n and we have an exponential expansion with the Hubble constant $H = (3t_2)^{-1}$. In the figure 1 we have plotted the time dependence of the scalar field, the scale factor and the deceleration parameter described by formula (I.24). From the graphs it is clearly seen the transition from the decelerated expansion to the accelerated phase.

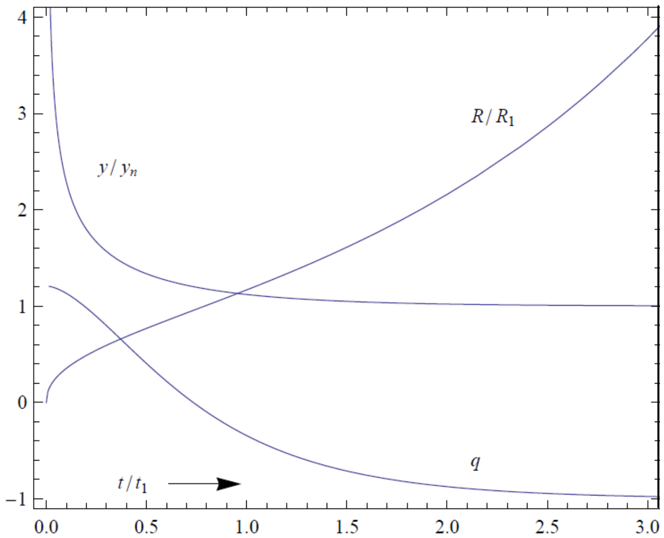

Fig.1. Scalar field, scale factor and the deceleration parameter as functions of the time for the value of the JBD theory parameter $\zeta = 2$.

Summarizing the analysis given above we conclude that if the non-minimally coupled scalar field dominates, then the decelerated expansion of the Universe is only possible as can be seen in Fig.1. Introducing the cosmological scalar in the case of nonminimally coupled scalar field, the Universe with transition from the phase with the decelerated expansion to the accelerated expansion phase is realized.

II. Scalar field in the Jordan-Brans-Dicke Theory and Dark Energy

We consider cosmological models in the framework of the Jordan-Brans-Dicke (JBD) theory in the Einstein frame. It separately discusses the cases of scalar field domination, as well as the presence of cosmological constant Λ, with matter, which is described by the barometric equation of state $P = \alpha\varepsilon$. The analysis of obtained results is carried out in the light of modern observational data. It is shown that the contribution of the scalar field and Λ-term $(\Lambda > 0)$ in the case of $q = -1/2$ (q is decelerating parameter) compensate each other, thus leading to Einstein's theory.

In the scales of $\sim 10^8$ light years and higher, the Universe can be considered as an isotropic and homogeneous structure, the matter of which is adequately described using the model of perfect fluid with the standard energy momentum tensor. In this work, we try to explain accelerating expansion of the Universe using a comparably simple, analytically built model. The problem is solved within the framework of JBD theory in the Einstein frame, in the presence of cosmological constant Λ.

As a result of variation of conformal transformed action of JBD theory [24]

$$W = \frac{1}{c}\int\sqrt{-\bar{g}}\left[-\frac{y_0}{2x}\left(\bar{R}+2\Lambda\right)+\frac{1}{2}\bar{g}^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta}+L_m\right]d^4x \tag{II.1}$$

field equations are presented in the following way:

$$\bar{g}^{\alpha\beta}\bar{\nabla}_{\alpha}\Phi_{,\beta} = 0 \tag{II.2}$$

$$\bar{G}_{\alpha\beta} - \Lambda\bar{g}_{\alpha\beta} = \left(x/y_0\right)\left(\bar{T}_{\alpha\beta}+\bar{\tau}_{\alpha\beta}\right), \tag{II.3}$$

$$\bar{\tau}_{\alpha\beta} = \Phi_{,\alpha}\Phi_{\beta} - \left(1/2\right)\bar{g}_{\alpha\beta}\bar{g}^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} \tag{II.4}$$

$$\Phi_{,\alpha} = \frac{y_{,\alpha}}{y}\sqrt{\frac{3+2\zeta}{2x}y_0}\ , \tag{II.5}$$

where y is the scalar potential of JBD theory. It is known that the Universe described by the geometry coincides with the metrics of Friedmann-Robertson-Walker (FRW). Thus, Eqs. (2)-(5) (in the units of $c = 1$) take the form

$$\frac{d}{dt}\left(\dot{\Phi}a^3\right) = 0, \qquad \dot{\Phi} = c_1\left(\frac{a_0}{a}\right)^3, \tag{II.6}$$

$$3\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right) = 8\pi G\left(\varepsilon+\frac{1}{2}\dot{\Phi}^2\right)+\Lambda, \tag{II.7}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2} = -8\pi G\left(\alpha\varepsilon+\frac{1}{2}\dot{\Phi}^2\right)+\Lambda, \tag{II.8}$$

$$\varepsilon(t) = \varepsilon_0\left(a_0/a\right)^n, \quad n = 3(1+\alpha), \tag{II.9}$$

$$P(t) = \alpha\varepsilon(t) \quad \text{c} \quad \alpha = -1, 0, 1/3, 1, \qquad \text{II.10)}$$

where the dot denotes time derivative a_0 is the scale factor $a(t)$ in the fixed moment of time t_0, and $c_1^2 = \left(2H_o^2/8\pi G\right)\left(1-q_o\right)-\varepsilon_o = \left(3H_o^2/8\pi G\right)\left[\left(2/3\right)\left(1-q_o\right)-\overset{o}{\Omega}_m\right]$, which in GR ($q_o = -1/2$ and $\overset{o}{\Omega}_m = 1$) leads to zero. Eqs. (II.6)-(II.10) are written more compactly, by using the Hubble constant

$H \equiv \dot{a}/a$ and the relation $\Omega(t) \equiv \varepsilon/\varepsilon_c$, where $\varepsilon_c \equiv 3H^2/8\pi G$, is introduced by analogy with the Einstein critical energy density $\overset{\mathrm{o}}{\varepsilon}_c = 3H_{\mathrm{o}}^2/8\pi G$.

(II.8) can be presented in the following way:

$$1 + \frac{k}{a^2H^2} = \frac{8\pi G}{3H^2}\varepsilon + \frac{8\pi G}{3H^2}\frac{\dot{\Phi}^2}{2} + \frac{\Lambda}{3H^2}, \quad 1 = \frac{\varepsilon}{\varepsilon_c} + \frac{\varepsilon_{ck}}{\varepsilon_c} + \frac{\varepsilon_\Lambda}{\varepsilon_c} - \frac{\varepsilon_k}{\varepsilon_c}, \tag{II.11}$$

where $\varepsilon_{ck} = (1/2)c_1(a_0/a)^6$, $\varepsilon_\Lambda = \Lambda/8\pi G$, $\varepsilon_k = 3k/8\pi G a^2$ are, respectively, energy of density of scalar field, generating Λ-term and field generating curvature of space.

Then in such a way, by introduction of so-called "decelerating" parameter $q = \ddot{a}a/\dot{a}^2$ as a result we get

$$2q + 1 = -3\alpha\frac{\varepsilon}{\varepsilon_c} - \frac{3\varepsilon_{ck}}{\varepsilon_c} + \frac{3\varepsilon_\Lambda}{\varepsilon_c} - \frac{\varepsilon_k}{\varepsilon_c}. \tag{II.12}$$

Therefore, in summary we get the compact system of equations

$$\Omega_m + \Omega_{ck} + \Omega_\Lambda = 1 + \Omega_k, \quad 2q + 1 = -3\alpha\Omega_m - 3\Omega_{ck} + 3\Omega_\Lambda - \Omega_k. \tag{II.13}$$

Here $\Omega_m \equiv \varepsilon/\varepsilon_c$, $\Omega_{ck} \equiv \varepsilon_{ck}/\varepsilon_c$, $\Omega_\Lambda \equiv \varepsilon_\Lambda/\varepsilon_c$, $\Omega_k \equiv \varepsilon_k/\varepsilon_c$. It is worth to write Eq. (II.13) also in the following way:

$$(3/2)\left(\dot{H}/H^2\right) + 1 = -\alpha\Omega_m - \Omega_{ck} + \Omega_\Lambda - (1/3)\Omega_k \tag{II.14}$$

from which it becomes apparent that when contributions of scalar field Ω_{ck} and Ω_Λ compensate each other, dynamics of the changing of H with time becomes like Einstein dynamics.
As a result we obtain

$$q = \Omega_\Lambda - 2\Omega_{ck} - (1 + 3\alpha)(\Omega_m/2), \tag{II.15}$$

from which follows that $\Omega_m = 1 - 2\Omega_{ck}$ for $q = -1/2$. Thus, in the framework of the considered model during the certain period of time, when $\Omega_{ck} = \Omega_\Lambda$, the Universe expands with deceleration, just as in Einstein's theory of Gravity. Then in a few years, the situation changes so much that q becomes positive [25]. It is natural enough to assume that at some intermediate moment of time, q becomes zero. According to our estimations it occurs when $\Omega_\Lambda \approx 0.52$ and $\Omega_{ck} \approx 0.18$, when $\Omega_m \approx 0.3$ as estimated in the set of works [26].

Let us try to find dynamical picture of time variation of the scale factor $a(t)$. In [27], exact analytical expressions of $a(t)$ are obtained for some cases of state equations. Let us rewrite these relations by taking into account the above-mentioned notations. For $q(t)$ it is convenient to use the formula

$$q=(3/2)(\Omega_\Lambda-\Omega_{ck})-1/2. \tag{II.16}$$

Scale factor $a(t)$ for the Universe with dust takes the following forms:

a) $\Lambda>0$. If the condition $4\overset{o}{\Omega}_{ck}\overset{o}{\Omega}_\Lambda>\left(\overset{o}{\Omega}_m\right)^2$ is satisfied, then

$$\left(\frac{a}{a_0}\right)^3=b^+\,\mathrm{sh}\left[3H_0\sqrt{\overset{o}{\Omega}_\Lambda}\,(t-t_0)+\delta^+\right]-\frac{1}{2}\left(\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right). \tag{II.17}$$

In the case of $4\overset{o}{\Omega}_{ck}\overset{o}{\Omega}_\Lambda<\left(\overset{o}{\Omega}_m\right)^2$,

$$\left(\frac{a}{a_0}\right)^3=b^-\,\mathrm{ch}\left[3H_0\sqrt{\overset{o}{\Omega}_\Lambda}\,(t-t_0)+\delta^-\right]-\frac{1}{2}\left(\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right). \tag{II.18}$$

For both circumstances, the symbol "o" denotes values corresponding to the fixed moment of time t_o. The constants have the following forms

$$\left(b^+\right)^2=\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}-\frac{1}{4}\left(\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right)^2, \tag{II.19}$$

$$\left(b^-\right)^2=\frac{1}{4}\left(\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right)^2-\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}, \tag{II.20}$$

$$e^{\delta^\pm}=\frac{1+\frac{1}{2}\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}+\sqrt{\left(1+\frac{1}{2}\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right)^2+b^{\pm\,2}}}{b^\pm}. \tag{II.21}$$

b) $\Lambda<0$. General solutions for $a(t)$ follow in this case from the equation

$$\frac{d}{dt}\left(\frac{a}{a_0}\right)^3=3H_0\sqrt{\overset{o}{\Omega}_\Lambda}\sqrt{\frac{\overset{o}{\Omega}_{ck}}{\left|\overset{o}{\Omega}_\Lambda\right|}+\frac{1}{4}\left(\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right)^2-\left[\left(\frac{a}{a_0}\right)^3-\frac{1}{2}\frac{\overset{o}{\Omega}_m}{\left|\overset{o}{\Omega}_\Lambda\right|}\right]^2}, \tag{II.22}$$

where the expression under the square root must be positive, i.e. only one of the possibilities for the Universe expansion can be realized:

$$\left(\frac{a}{a_0}\right)^3>\frac{1}{2}\frac{\overset{o}{\Omega}_m}{\left|\overset{o}{\Omega}_\Lambda\right|}+\sqrt{\frac{\overset{o}{\Omega}_{ck}}{\left|\overset{o}{\Omega}_\Lambda\right|}+\frac{1}{4}\left(\frac{\overset{o}{\Omega}_m}{\overset{o}{\Omega}_\Lambda}\right)^2} \tag{II.23}$$

The solution of (II.22) has the form

$$\left(\frac{a}{a_0}\right)^3=\sqrt{\frac{\mathring{\Omega}_{ck}}{\left|\mathring{\Omega}_\Lambda\right|}+\frac{1}{4}\left(\frac{\mathring{\Omega}_m}{\mathring{\Omega}_\Lambda}\right)^2}\,\sin\left(3H_0\sqrt{\mathring{\Omega}_\Lambda}\,t\right)\frac{1}{2}\frac{\mathring{\Omega}_m}{\left|\mathring{\Omega}_\Lambda\right|}\,. \tag{II.24}$$

Now we can see what can be obtained for the age of the Universe. For the light coming from a stellar object, redshift is typical, which is conditioned by the expansion of the Universe. The wavelength increases linearly proportional to the scale factor $a(t)$. This effect can be taken into account by introducing the concept of redshift z:

$$1+z=\frac{\lambda_o}{\lambda}=\frac{a_o}{a}, \tag{II.25}$$

where index "o" corresponds to the moment of observation. From Eq. (II.25) it follows

$$\dot{z}=-H\left(1+z\right), \tag{II.26}$$

which makes it possible to evaluate the age of the Universe

$$\Delta t_u=\int_0^{t_0}dt=\int_0^{\infty}\frac{dz}{H\left(1+z\right)}\,. \tag{II.27}$$

From Eqs. (II.15) and (II.27) we get

$$\Delta t_u=\frac{1}{H_o}\int_o^{\infty}\frac{dz}{\left(1+z\right)\sqrt{\mathring{\Omega}_{ck}\left(1+z\right)^6+\mathring{\Omega}_m\left(1+z\right)^3-\mathring{\Omega}_k\left(1+z\right)^2+\mathring{\Omega}_\Lambda}}\,. \tag{II.28}$$

Replacement of the variable by $y\equiv 1/\left(1+z\right)^3$ in the case of the conventional conception of a flat Universe gives the integral of the following form

$$\Delta t_u=\frac{1}{3H_o}\int_0^{1}\frac{dy}{\sqrt{\mathring{\Omega}_\Lambda y^2+\mathring{\Omega}_m y+\mathring{\Omega}_{ck}}}\,. \tag{II.29}$$

In the case of the values of $\Omega_\Lambda=0,\ \mathring{\Omega}_{ck}=0,\ \mathring{\Omega}_m=1$ Eq. (II.29) gives the Einstein's estimation of the age of the Universe:

$$\Delta t_U^a=\frac{2}{3}H_o^{-1}\approx 8\div 10\ \text{Gyr}, \tag{II.30}$$

where we used value of H according to the Hubble Space Telescope Key Project $H_o^{-1}=9.77\times h^{-1}$ Gyr, $0.64<h<0.8$. This is not in agreement with the limited value of stellar lifetime, which is $>11\div 12$ Gyr. Thus in GR there exist the problem of age.

In our case, the integral (II.27) is equal to

$$\Delta t_u=\frac{2}{3H_o 2\sqrt{\mathring{\Omega}_\Lambda}}\ln\left(\frac{2\sqrt{\mathring{\Omega}_\Lambda}+2\mathring{\Omega}_\Lambda+\mathring{\Omega}_m}{2\sqrt{\mathring{\Omega}_\Lambda\mathring{\Omega}_{ck}}+\mathring{\Omega}_m}\right)=\frac{\Delta t_U^a}{2\sqrt{\mathring{\Omega}_\Lambda}}\ln\left(\frac{2\sqrt{\mathring{\Omega}_\Lambda}+2\mathring{\Omega}_\Lambda+\mathring{\Omega}_m}{2\sqrt{\mathring{\Omega}_\Lambda\mathring{\Omega}_{ck}}+\mathring{\Omega}_m}\right). \tag{II.31}$$

In the case of values $\Omega_\Lambda = \Omega_{ck}$, the age of the Universe is close to the estimate gotten in GR:

$$\Delta t_u = \Delta t_U^e \ln\left(1 + 2\sqrt{\mathring{\Omega}_\Lambda}\right) \Big/ 2\sqrt{\mathring{\Omega}_\Lambda}\,. \tag{II.32}$$

The standard cosmological model is investigated within the framework of the Einstein frame of JBD theory, from the viewpoint of the contribution of different components' energy densities. It is shown that for the value $q = -1/2$ (estimated in the WMAP experiment [28]) contributions of energies, conditioned by the presence of scalar field and Λ-term, compensate each other and the expansion occurs by the scheme like that of Einstein's. In the future, when this condition is violated, q becomes zero for the values $\Omega_m \approx 0.3$, $\Omega_\Lambda = 0.52$, $\Omega_{ck} = 0.18$, after which it becomes positive.

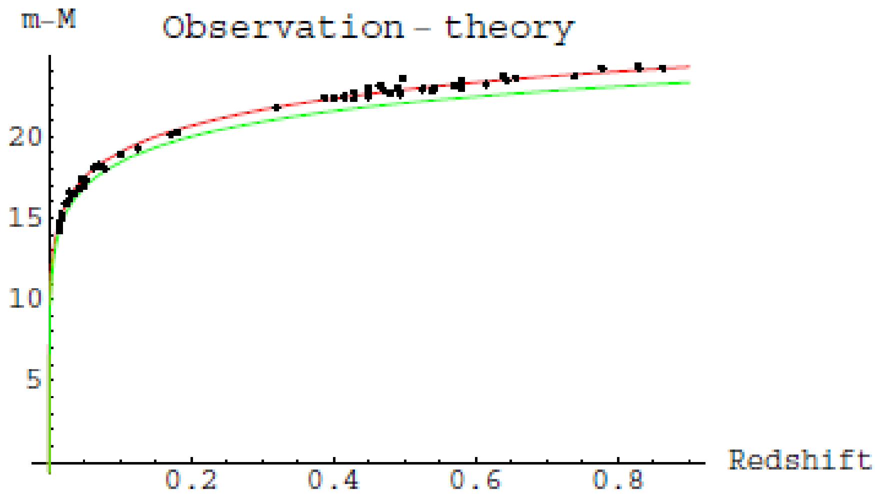

Fig.2. The black points are the observational data from Ia type Supernovae. Green is the theoretical curve for the case $\Omega_\Lambda = 0.65$. Red is the theoretical curve for the case $\Omega_\Lambda = 0.5$. On *y*-axis is the effective stellar magnitude $m - M = 5\log_{10}\left(d_L/Mpc\right) + 25$.

As a result of this work, it is worthwhile to present time dependence of $a(t)$, and also $H(t)$, to qualitatively describe dynamics of the Universe evolution in the limited case of minimally-coupled scalar and tensor fields. As can be seen in Fig.2, the theoretical curve is closer to the observational data in the case of bigger values of $\mathring{\Omega}_\Lambda$. Here we used observational data of Ia type Supernovae [28] and show the dependence of effective stellar magnitude on *z* redshift [29]. According to our cosmological model, the phase transition of the Universe expansion from a decelerating to an accelerating one can be realized as shown in Fig.3. One can also see the behavior of the Hubble Rate. For a more complete picture of the Universe Evolution, we also present the dependence on time for the different contributions, in Fig.4; red curve represents $\Omega_{ck}(t)$, green curve represents $\Omega_m(t)$, and blue curve represents $\Omega_\Lambda(t)$. From this figure, the growth of $\Omega_\Lambda(t)$ is obvious.

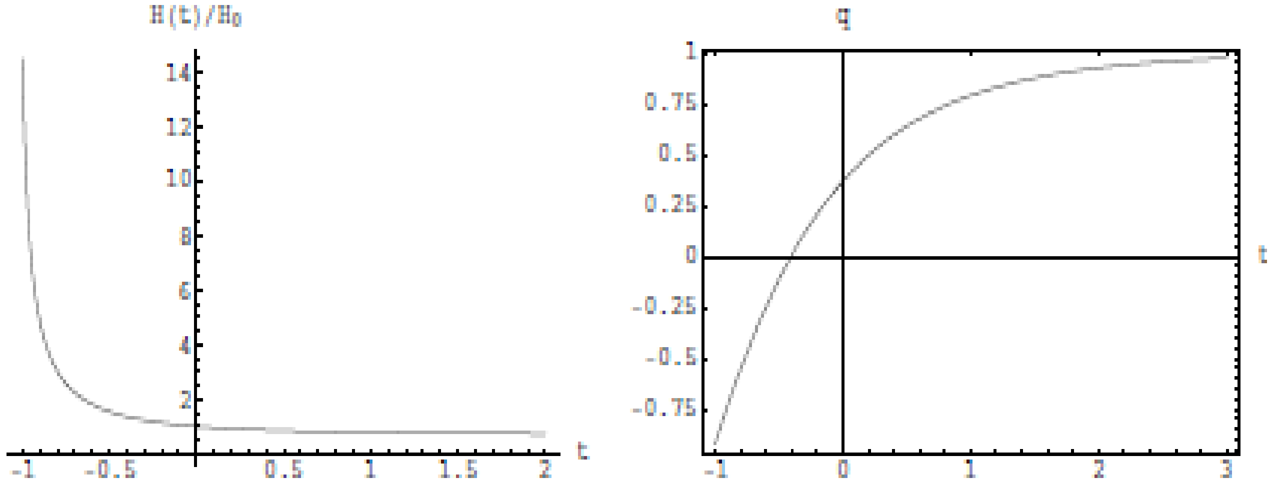

Fig.3. Left figure is theoretical curve for Hubble Rate, and the right figure is theoretical curve of "deceleration" parameter. For both cases, t is in units of $3H_0\sqrt{\overset{\circ}{\Omega}_\Lambda} \approx (4\div 5)\sqrt{\overset{\circ}{\Omega}_\Lambda}$ Gyr, and the zero point corresponds to the moment of observation.

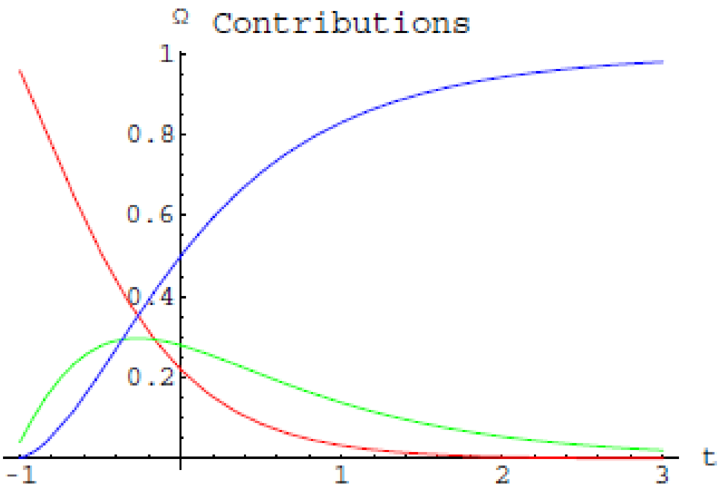

Fig.4. Red curve corresponds to $\Omega_{ck}(t)$. Green curve corresponds to $\Omega_m(t)$ and blue curve corresponds to the $\Omega_\Lambda(t)$. t is in units of $3H_0\sqrt{\overset{\circ}{\Omega}_\Lambda} \approx (4\div 5)\sqrt{\overset{\circ}{\Omega}_\Lambda}$ Gyr, and the zero point corresponds to the moment of observation.

III. Radiation Dominated Epoch with a Nonminimally-Coupled Scalar Field

Let us consider the cosmological problem again corresponding to the FWR metrics for a flat space and $R(t)$ is the scale factor, the cosmological equation in the proper frame of the JBD theory can be written in the form (a dot represents the time derivative) [30]

$$\frac{1}{R^3}\frac{d}{dt}\left(\dot{y}R^3\right) = \frac{\kappa\left(1-3\alpha\right)}{\left(3+2\zeta\right)}\varepsilon\,, \tag{III.1}$$

$$3\left(\frac{\dot{R}^2}{R^2}\right) = \frac{\kappa\varepsilon}{y} - \frac{3\dot{R}\dot{y}}{Ry} + \frac{\zeta}{2}\frac{\dot{y}^2}{y^2} + \frac{y}{y_0}\Lambda\,, \tag{III.2}$$

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=-\frac{\kappa p}{y}-\frac{\ddot{y}}{y}-\frac{2\dot{R}\dot{y}}{Ry}-\frac{\zeta}{2}\frac{\dot{y}^2}{y^2}+\frac{y}{y_0}\Lambda, \qquad \text{(III.3)}$$

with

$$\dot{\varepsilon}=-3\frac{\dot{R}}{R}(\varepsilon+p), \qquad \varepsilon=\frac{\varepsilon_0}{R^4}. \qquad \text{(III.4)}$$

Here $p=\alpha\varepsilon$ and $\alpha=1/3$..

Since (III.1) implies that

$$\dot{y}R^3=D=\text{const.}$$

It is natural to introduce the quantity $y=Df(t)/R^2$ and then proceed to differentiation with respect to $\eta\,(d\eta=dt/R)$, to obtain

$$2\dot{R}f=f'-1, \qquad \text{(III.5)}$$

where the prime denotes differentiation with respect to η .

Using Eqs. (III.4) and (III.5), Eq. (III.2) can be written in the form

$$\dot{R}^2f^2+\dot{R}f-\left(\frac{\kappa\varepsilon_0}{3D}f+\frac{\Lambda D}{3y_0}f^3+\frac{\zeta}{6}\right)=0. \qquad \text{(III.6)}$$

Solving this quadratic form for $\dot{R}f$,

$$\dot{R}f=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+\frac{\kappa\varepsilon_0}{3D}f+\frac{\Lambda D}{3y_0}f^3+\frac{\zeta}{6}}, \qquad \text{(III.7)}$$

and using Eq. (III.5), we obtain an equation for the function $f(\eta)$:

$$f'=\pm\sqrt{1+\frac{2\zeta}{3}+\frac{4\kappa\varepsilon_0}{3D}f+\frac{4\Lambda D}{3y_0}f^3}. \qquad \text{(III.8)}$$

If we introduce the indefinite multipliers α and β, where

$$f=\alpha F, \quad \tau=\frac{\eta}{\beta}$$

and choose them so that

$$\alpha^2=\frac{4k\varepsilon_0y_0}{\Lambda D^2}, \quad \beta^4=\frac{9y_0}{4k\varepsilon_0\Lambda},$$

then Eq. (III.8) can be reduced to

$$\frac{dF}{d\tau}=\pm\sqrt{\frac{\beta^2}{\alpha^2}\left(1+\frac{2}{3}\zeta\right)+F+\frac{F^3}{4}}, \frac{\beta^2}{\alpha^2}=\frac{8}{3D^2}\sqrt{k^3\varepsilon_0^3\frac{y_0}{\Lambda}}, \qquad \text{(III.9)}$$

where F is the Weierstrass function [31].

Decelerating parameter in the proper frame of the JBD theory

Equation (III.5) implies that

$$\frac{R'}{R} = \dot{R} = \frac{(\alpha/\beta)\,dF/d\tau - 1}{2\alpha F},$$

so that $R''/R - R'^2/R^2 = f''/f - (f'-1)\,f'/2f^2$ and the decelerating parameter $q = \ddot{R}R/\dot{R}^2$ [30] takes the form

$$q = \frac{R''R}{R'^2} - 1 = \frac{2ff'' - 2f'^2 + f'}{(f'-1)^2}. \tag{III.10}$$

Using Eq. (III.8), we obtain

$$f'' = \frac{2}{3}\left(\frac{k\varepsilon_0}{D} + \frac{3\Lambda D}{y_0} f^2\right), \tag{III.11}$$

so that

$$q = \frac{(2/3)\left(k\varepsilon_0/D + (3\Lambda D/y_0)\,f^2\right)}{(f'-1)^2} - \frac{2f'}{f'-1}. \tag{III.12}$$

The results of some numerical calculations of $q(t)$ for this radiation epoch are shown in Fig.5, where curve 1 corresponds to $\zeta = 1$, curve 2 corresponds to $\zeta = 100$ and curve 3 to $\zeta = 1000$. A trend in the behavior of the maximum positive value, $q_{\max}$, can be seen clearly, while $q \to +1$ in the current stage of the Universe evolution.

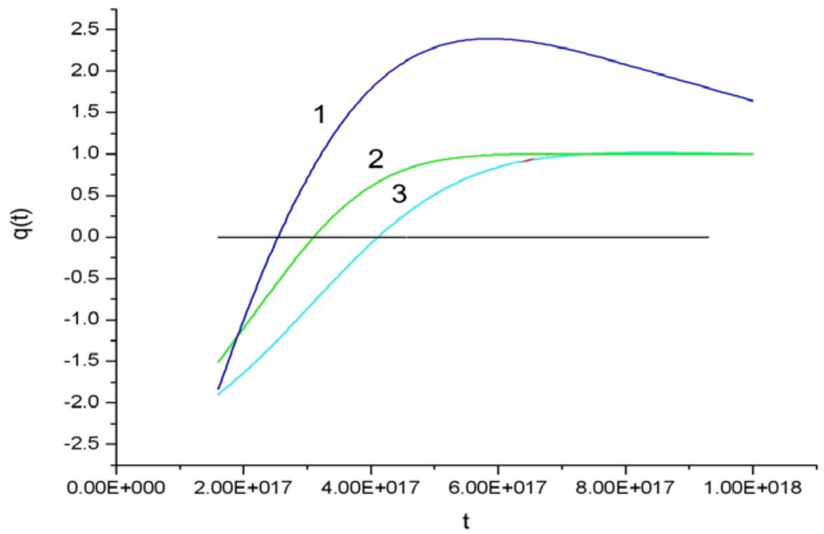

Fig.5. The deceleration parameter q as a function of time.

As a result taking into account gravitational scalar leads to $q > 0$ in the radiation dominated epoch too.

IV Cosmological Problem with Conformaly Coupled Scalar Field

In the case of conformly coupled scalar field in the presence of Higgs potential, the action has the following form [12,32]:

$$W=\int\left[-\left(\frac{1}{2k}-\frac{\psi^2}{12}\right)\rightleftharpoons R+\frac{1}{2}(\nabla\psi)^2-\frac{\lambda}{12}\psi^4+L_m\right]\rightleftharpoons\sqrt{-g}d^4x\,. \tag{IV.1}$$

As a result of the variation over g_{ik} and ψ we obtain the following field equations:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=k\left(1-\frac{k\psi^2}{6}\right)^{-1}\left[\tau_{\mu\nu}+T_{\mu\nu}^{mat}\right], \tag{IV.2}$$

$$\nabla^2\psi-\frac{R}{6}\psi+\frac{\lambda}{3}\psi^3=0, \tag{IV.3}$$

where

$$\tau_{\mu\nu}=\frac{2}{3}\nabla_\mu\psi\nabla_\nu\psi-\frac{1}{6}g_{\mu\nu}(\nabla\psi)^2-\frac{\psi}{3}\nabla_\mu\nabla_\nu\psi+\frac{\psi}{3}g_{\mu\nu}\nabla^2\psi+\frac{\lambda}{12}\psi^4g_{\mu\nu}, \tag{IV.4}$$

$$T_{\mu\nu}^{mat}=(\varepsilon+P)U_\mu U_\nu-Pg_{\mu\nu},$$

In this case, the contraction of (2.2), by taking into account (2.3), gives

$$-R=kT=k(\varepsilon-3P) \tag{IV.5}$$

It is commonly assumed that the geometry with the Friedmann–Robertson–Walker metrics is adequate for the model of homogeneous and isotropic Universe. Accordingly, for the flat model with the matter having the equation of state $P=\alpha\varepsilon$ the above equations become

$$\frac{\dot{R}^2}{R^2}=\frac{k}{3\left(1-k\psi^2/6\right)}\left[\frac{\dot{\psi}^2}{2}-\psi\dot{\psi}\frac{\dot{R}}{R}+\varepsilon\right] \tag{IV.6}$$

$$-\left(\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)=\frac{k}{3\left(1-k\psi^2/6\right)}\left[\frac{\dot{\psi}^2}{2}-2\frac{\dot{R}}{R}\psi\dot{\psi}-\psi\ddot{\psi}+3P\right], \tag{IV.7}$$

$$\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=\frac{k}{6}(\varepsilon-3P), \tag{IV.8}$$

$$\ddot{\psi}+3\frac{\dot{R}}{R}\dot{\psi}+\psi\left(\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)+\frac{\lambda}{3}\psi^3=0\,, \tag{IV.9}$$

$$\dot{\varepsilon}=-\left(3\dot{R}/R\right)(\varepsilon+P),\quad \varepsilon=\varepsilon_0/R^n,\quad n=3(1+\alpha)\,. \tag{IV.10}$$

In the system (IV.6-IV.10) one of the equations is a consequence of the others.

From (IV.8) we obtain for the deceleration parameter $q=R\ddot{R}/\dot{R}^2$ the following expression:

$$q=-1+\left(k/6H^2\right)(\varepsilon-3P), \tag{IV.11}$$

where $H=\dot{a}/a$ is the Hubble's parameter, which allows to estimate the possibility of uniform or accelerated expansion of the Universe, because it follows from the condition $q\geq 0$ that

$$\varepsilon\geq 2\frac{3H^2}{8\pi G}+3P \tag{IV.12}$$

(Recall that within the framework of Einstein's theory $\varepsilon_{crit} = 3H^2/8\pi G$.)

For the values of α, typical in cosmology, the factor q is expressed as follows:

$$\begin{aligned}
q &= -1 + \frac{2k\varepsilon}{3H^2}, && \text{vacuum model } (\alpha = -1),\\
q &= -1 + \frac{k\varepsilon}{6H^2}, && \text{matter dominated } (\alpha = -1),\\
q &= -1, && \text{radiation dominated } (\alpha = 1/3),\\
q &= -1, && \text{scalar field dominated } (\varepsilon = 0,\ P = 0),\\
q &= -1 - \frac{k\varepsilon}{3H^2}, && \text{stiff fluid } (\alpha = -1).
\end{aligned} \tag{IV.13}$$

Using (IV.8), the Hubble parameter can be presented in the following form:

$$H^2 = \frac{k\varepsilon_0}{3a^{3(1+\alpha)}} + \frac{\beta}{R^4},$$

with the integration constant β.

a) If the scalar field dominates $(\varepsilon = 0,\ P = 0)$ or in the case of $P = \varepsilon/3$, the scalar curvature goes to zero $(R = 0)$, and as a result $R = \sqrt{2ct + b},\ H = c/R^2,\ \psi = c_2 - c_1/cR$, with

$$c^2\left(3 - c_2^2 k/2\right) = 0 \Rightarrow c_2^2 = 6/k\,, \tag{IV.14}$$

$$\psi = \sqrt{6/k} - c_1/ca,$$

i.e. at sufficiently large a we have $\psi \to \sqrt{6/k}$.

b) If the radiation dominates $(\alpha = 1/3)$, in the presence of scalar field, the analytic forms for the functions $R,\ \psi,\ H$ remain the same, and condition (IV.14) becomes

$$c^2\left(3 - c_2^2 k/2\right) = k\varepsilon_0, \tag{IV.15}$$

so now $c_2 = \sqrt{6/k - 2\varepsilon_0/c^2}$.

c) The matter dominated era corresponds to $P = 0,\ \varepsilon = \varepsilon_0/R^3$, therefore for $H(R) = \dot{R}/R$ we obtain

$$H^2 = \frac{1}{R^4}\left(\frac{k\varepsilon_0}{3}R + \beta\right) = \frac{k\varepsilon}{3} + \frac{\beta}{R^4} \tag{IV.16}$$

and, correspondingly,

$$q = -1 + \frac{k\varepsilon_0\, R/6}{k\varepsilon_0\, R/3 + \beta}. \tag{IV.17}$$

It follows from (IV.17) that $q \to -1/2$ at $R \to \infty$ (as in GR), and the requirement $q \geq 0$, taking into account the positivity of (IV.16) and the condition $\beta < 0$ is reduced to the following inequality:

$$1 \leq v \leq 2\,,$$

where $v \equiv \left(k\varepsilon_0/3|\beta|\right)R(t)$.

Thus, the time evolution of the scale factor is determined from the following equation:

$$0 < H = \frac{\dot{v}}{v} = \frac{A\sqrt{v-1}}{v^2}, \tag{IV.18}$$

with $A \equiv \left(k\varepsilon_0/3\right)^2 |\beta|^{-3/2}$.

As a result of integration, the following cubic equation is obtained:

$$\frac{2z^3}{3} + 2z = A\left(t - t_0\right), \quad z = \sqrt{v-1}.$$

The solution of this cubic equation can be obtained using the Cardano formula [33] and the solution has the following form:

$$z = 2\sinh\left[\frac{1}{3}\operatorname{arcsinh}\left(\frac{3A}{4}\left(t-t_0\right)\right)\right], \tag{IV.19}$$

from which one finds

$$t_0 \le t \le t_0 + \frac{24|\beta|^{3/2}}{\left(k\varepsilon_0\right)^2} \tag{IV.20}$$

To determine the time evolution of the scalar field ψ, let us consider (2.9) in the case $\lambda = 0$. The substitution $U = a\psi$ leads to the equation

$$\ddot{U} - \dot{U}\frac{\dot{R}}{R} = 0, \tag{IV.21}$$

and as a result

$$\psi = \gamma\dot{R} - \delta/R, \tag{IV.22}$$

where γ and δ are constant.

d) The system

$$\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = \frac{2k\varepsilon}{3}, \tag{IV.23}$$

$$\ddot{\psi} + 3\sqrt{\frac{k\varepsilon_0}{3} + \frac{\beta}{a^4}}\dot{\psi} + \frac{2k\varepsilon_0}{3}\psi = 0, \tag{IV.24}$$

corresponds to the equation of state $P = -\varepsilon$.

By the substitution $v = R^2$, (IV.23) is reduced to

$$\ddot{v} - \frac{4k\varepsilon}{3}v = 0 \quad \text{or} \quad \dot{v}^2 - \frac{4k\varepsilon}{3}v^2 = \beta, \tag{IV.25}$$

the solution of which can be presented in the following form:

$$\frac{R^2(t)}{R^2(0)} = \cosh\left(2\sqrt{\frac{k\varepsilon}{3}}t\right) + \sqrt{\frac{3}{k\varepsilon}}H(0)\sinh\left(2\sqrt{\frac{k\varepsilon}{3}}t\right) \tag{IV.26}$$

Accordingly, (IV.24) is reduced to the equation

$$\psi''_{\tau\tau} + \frac{3}{2}f(\tau)\psi'_\tau + \frac{1}{2}\psi = 0\,, \tag{IV.27}$$

where the derivatives are taken with respect to $\tau = 2\sqrt{k\varepsilon/3}\,(t - t_0)$, and $f = \tanh\tau$ or $f = \coth\tau$ depending on the choice of the initial conditions. Solution (IV.27) is expressed in terms of the hypergeometric function. In a special case of sufficiently large $R(t)$ or for $\beta = 0$, (IV.27) is reduced to

$$\ddot{\psi} + 3\sqrt{\frac{k\varepsilon_0}{3}}\dot{\psi} + \frac{2k\varepsilon_0}{3}\psi = 0,$$

and as a result ψ becomes, correspondingly,

$$\psi = \frac{c_1}{R^2} + \frac{c_2}{R} \quad \text{or} \quad \psi = c_1 e^{-2\sqrt{k\varepsilon_0/3t}} + c_2 e^{-\sqrt{k\varepsilon_0/3t}}$$

so that $R = e^{\sqrt{k\varepsilon_0/3t}}$ and $q = 1$.

4. Conclusion

Modern cosmological observations claim an accelerated expansion of the Universe in the present epoch. To explain this in the framework of GR, the presence of dark energy or of some non-gravitational source which satisfies the condition $\varepsilon + 3P < 0$ (ε – energy density, P – pressure) is needed. The cosmological constant is the simplest candidate for this role, but its numerical value contains unsolved problematic questions. This has led to raising many alternatives to GR, particularly the scalar-tensor theories of gravity. In the present paper we reviewed a series of works, based on modified versions of JBD theory. In the section I of this work the cosmological model with non-minimally coupled scalar field is considered in the presence of cosmological scalar $\varphi = y\Lambda/y_0$ in the proper frame of JBD theory, however cosmological scalar is chosen so as in the Einstein frame, it becomes a cosmological constant. It is shown that in this case phases with the unique and then with accelerated expansion appears in the scale of cosmological times.

The corresponding action has the form (I.1) and we have considered a special case of the function $\varphi(y) = y\Lambda/y_0$. With this choice the gravitational part of the JBD action in the Eisntein frame coincides with the cosmological expansion in the Jordan frame is always decelerated. When the cosmological scalar is present in the case of non-minimally coupled scalar field, the Universe with transition from the phase of the decelerated expansion to the accelerated expansion phase is realized. At late stages we have an exponential expansion and the scalar field tends to constant value. This behavior is clearly seen on the example presented in Fig. 1. In the case of minimally coupled scalar field the situation is basically the same.

In section II, a cosmological model of the Universe is considered in the framework of the Einstein frame of JBD theory in the presence of the cosmological constant. The analysis is done for obtained analytical results in accordance with modern observational data. Comparable plots of an absolute magnitude vs. redshift on the base of observational data are presented, as well as theoretical curve of “decelerating” q parameter, curves of energy contributions of minimally coupled scalar field, of normal matter and of non-gravitational field in the case of the flat Universe. From the presented plots one can see the “deceleration” parameter's transition through zero, thus the phase transition from decelerating to accelerating expansion near the present time.

In section III model of unique and isotropic Universe is considered in the radiation dominated epoch. Results of numerical calculations for $q(t)$ for the different values of JBD theory parameter are presented in the plot. It is shown that in this stage the positive acceleration is possible.

In the last section, the action is given by expression (IV.1) which describes a conformal coupled scalar field with quadratic potential. As an additional gravitational source we have considered matter with barotropic equation of state (see (IV.10)). The conditions are specified under which the cosmological expansion contains an accelerated phase. In particular, we have shown that if the scalar field dominates or in the case of the radiation type matter the expansion is always decelerating. In the case of dust matter two classes of models are realized. In the first one, corresponding to positive values of the integration constant β in (IV.16), the expansion is always decelerated with standard GR solution as late time attractor. The second class of models is realized for negative values of β. In this case the expansion of the Universe starts from a finite value of the scale factor and at the early stages we have an accelerated expansion with the subsequent transition to the decelerated phase with scalar field testing to zero at late stages. When the additional source in the cosmological equations is described by the cosmological constant, the early-time evolution essentially depends on the sign of the integration constant β in the right-hand side of (IV.24). The corresponding scale factor is given by formulae (IV.25) and (IV.26) and has minimum nonzero value for negative values of β. At late stages of the cosmological expansion the scalar field vanishes exponentially and we have exponential expansion driven by the cosmological constant.

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