IBM-1 INVESTIGATION FOR THE EVEN-EVEN $^{180-190}$W ISOTOPES

Mohammed Abdul Ameer, Mushtaq A. H. Al-Shimmary

Department of Physics, College of Science, University of Babylon, P.O.Box 4, Babil-IRAQ, e-mail: mushtaq.alshimmary@yahoo.com

Received 17 July, 2011

Abstract—The $^{180-190}$W isotopes in SU(3)-O(6) transition region were investigated. For these nuclei, the energy levels, B(E2) transition probabilities, and electric quadrupole moment $Q^{(2)}_{\pm}$ were calculated within framework of the Interacting Boson Model (IBM-1). The results are compared with the most recent experimental data. Good agreement was obtained between our theoretical calculations for all isotopes under study.

Keywords: energy levels, transition probabilities, quadrupole moment

1. Introduction

The neutron-proton interaction is known to play a dominant role in quadrupole correlations in nuclei. As a consequence, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell.

The interacting boson model offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, based on general algebraic group theoretical techniques which have also recently found application in problems in atomic, molecular, and high-energy physics [1,2].

The even-mass tungsten isotopes have been previously investigated both theoretically and experimentally in recent years with special emphasis on interpreting experimental data via collective models. Energy levels, electric quadrupole moments, B(E2) values of $^{182-186}$W isotopes have been studied within the framework of the interacting boson model IBM-2 [3], the nuclear structure of $^{182-186}$W isotopes have been investigated by J.B.Gupta [4] using the IBM-1 model.

Our aim in this study is to investigate even-even $^{180-190}$W isotopes in SU(3)-O(6) transition region and calculate the energy levels, B(E2) transition probabilities, and electric quadrupole moment $Q^{(2)}_{\pm}$ within framework of the Interacting Boson Model (IBM-1).

2. The Interacting Boson Model

The interacting boson model of Arima and Iachello [5-10] has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. In this model it was assumed that low-lying collective states of even-even nuclei could be described as states of a given (fixed) number $N$ of bosons. Each boson could occupy two levels one with angular momentum $L = 0$ ($s$-boson) and another, usually with higher energy, with $L = 2$ ($d$-boson). In the original form of the model known as IBM-1, proton-
and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure, associated with it. In terms of $s$- and $d$-boson operators the most general IBM Hamiltonian can be expressed as [11]

$$
H = \tilde{\epsilon}_d \hat{n}_d + \frac{1}{2} \sum_J C_J (d^\dagger d^J + d^J d^\dagger) + \frac{v_2}{\sqrt{10}} \left( (d^\dagger d^2)^{(2)} (\tilde{d} s) + \text{H.c.} \right) + \frac{v_0}{2\sqrt{5}} \left( (d^\dagger d^2)^{(2)} (ss)^{(0)} + \text{H.c.} \right).
$$

This Hamiltonian contains 2 one-body term, ($\tilde{\epsilon}_s$ and $\tilde{\epsilon}_d$), and 7 two-body interactions ($c_J (J = 0, 2, 4)$, $v_J (J = 0, 2)$, $u_J (J = 0, 2)$), where $\tilde{\epsilon}_s$ and $\tilde{\epsilon}_d$ are the single-boson energies, and $c_J$, $v_J$, $u_J$ describe the two-boson interactions. However, it turn out that for fixed boson number $N$, only one of the one-body terms and five of the two-body terms are independent, as it can be seen by noting $N = n_s + n_d$. Hamiltonian (1) can be rewritten in terms of the Casimir operators of $\text{U}(6)$ group. In that case, one says that the Hamiltonian $H$ has a dynamical symmetry. These symmetries are called SU(5) vibrational, SU(3) rotational and O(6) $\gamma$-unstable.

However, it is more common to write the Hamiltonian of the IBM-1 as a multipole expansion, grouped into different boson-boson interactions Eq. (1) [12]:

$$
H = \tilde{\epsilon}_n \hat{n}_n + a_0 \hat{P}^\dagger \cdot \hat{P} + a_2 \hat{J} + a_2 \hat{Q} + a_4 \hat{T}_3 + a_4 \hat{T}_4.
$$

The operators are defined by the following equations:

$$
\hat{n}_n = (\hat{d}^\dagger \cdot \hat{d}), \quad \hat{P}^\dagger = \frac{1}{2} (\hat{d}^\dagger \cdot \hat{d} - \hat{s}^\dagger \cdot \hat{s}),
$$
$$
\hat{J} = \sqrt{10} \left[ \hat{d}^\dagger \times \hat{d} \right]^{(0)},
$$
$$
\hat{Q} = \left[ \hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d} \right]^{(2)} + \chi \left[ \hat{d}^\dagger \times \hat{d} \right]^{(2)},
$$
$$
\hat{T}_3 = \left[ \hat{d}^\dagger \times \hat{d} \right]^{(3)}, \quad \hat{T}_4 = \left[ \hat{d}^\dagger \times \hat{d} \right]^{(4)}.
$$

The E2 transition operator must be a Hermitian tensor of rank two and therefore the number of bosons must be conserved. Since, with these constraints there are two operators possible in the lowest order, the general E2 operator can be written as [13]

$$
\hat{T}_\mu^{(E2)} = \alpha_2 \left[ \hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d} \right]^{(2)} + \beta_2 \left[ \hat{d}^\dagger \times \hat{d} \right]^{(2)},
$$

where $\alpha_2$ plays the role of the effective boson charge and $\beta_2 = -\sqrt{7}/2\alpha_2$. The B(E2) strength for the E2 transitions is given by

$$
B(E2) = \alpha_2 \left[ \hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d} \right]^{(2)} + \beta_2 \left[ \hat{d}^\dagger \times \hat{d} \right]^{(2)}.
$$
3. Results and Discussion

3.1. Energy Levels

The examination of the experimental energy levels for the nuclei $^{180-190}$W shows that they lie in the transitional region SU(3)$\rightarrow$O(6), therefore the Hamiltonian of the transition region SU(3)$\rightarrow$O(6) has been employed in the calculation by using the program PRINT written by O. Scholten [14]. The best fit for the Hamiltonian parameters Eq. (2) used in the present work are presented in Table 1 which gives the best agreement between the calculated energy levels in the present work and their corresponding experimental data taken from [15] as shown in Figs. 1-6.

![Fig. 1. The experimental and theoretical energy levels for $^{180}$W.](image1)

![Fig. 2. The experimental and theoretical energy levels for $^{182}$W.](image2)
Fig. 3. The experimental and theoretical energy levels for $^{184}\text{W}$.

Fig. 4. The experimental and theoretical energy levels for $^{186}\text{W}$. 
Fig. 5. The experimental and theoretical energy levels for $^{188}$W.

Fig. 6. The experimental and theoretical energy levels for $^{190}$W.

Table 1. The Hamiltonian parameters used in the IBM-code (PHINT) for $^{180}$-$^{190}$W isotopes

<table>
<thead>
<tr>
<th>$A$</th>
<th>$N$</th>
<th>EPS (MeV)</th>
<th>$\hat{\rho}$ (MeV)</th>
<th>$\hat{j} \cdot \hat{j}$ (MeV)$^2$</th>
<th>$\hat{Q} \cdot \hat{Q}$ (MeV)$^2$</th>
<th>$\hat{T}_x \cdot \hat{T}_x$ (MeV)$^2$</th>
<th>CHI</th>
<th>SO6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{180}$W</td>
<td>14</td>
<td>0.0000</td>
<td>0.05520</td>
<td>0.01149</td>
<td>$-0.01380$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>$-1.3228$</td>
</tr>
<tr>
<td>$^{182}$W</td>
<td>13</td>
<td>0.0000</td>
<td>0.05980</td>
<td>0.01107</td>
<td>$-0.01495$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>$-1.3228$</td>
</tr>
<tr>
<td>$^{184}$W</td>
<td>12</td>
<td>0.0000</td>
<td>0.04591</td>
<td>0.01422</td>
<td>$-0.01147$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>$-1.3228$</td>
</tr>
<tr>
<td>$^{186}$W</td>
<td>11</td>
<td>0.0000</td>
<td>0.03906</td>
<td>0.01677</td>
<td>$-0.00976$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>$-1.3228$</td>
</tr>
<tr>
<td>$^{188}$W</td>
<td>10</td>
<td>0.0000</td>
<td>0.03417</td>
<td>0.02062</td>
<td>$-0.00854$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>$-1.3228$</td>
</tr>
<tr>
<td>$^{190}$W</td>
<td>9</td>
<td>0.0000</td>
<td>0.02055</td>
<td>0.03257</td>
<td>$-0.00513$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>$-1.3228$</td>
</tr>
</tbody>
</table>
3.2. Reduced transition probabilities $B(E2)$ and Quadrupole moment $Q$

More information can be obtained by studying the reduced transition probabilities $B(E2)$. The (IBMT-code) were employed and the values of $(\alpha_2, \beta_2)$ were estimated to reproduce the experimental $B(E2; 2^-_1 \rightarrow 0^+_1)$ values. The parameters $E2SD$ and $E2DD$ used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in Table 2.

Table 2. The experimental values of $B(E2)$ and the coefficients($E2SD$, $E2DD$) for $^{180-190}$W used in the present work.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B(E2; 2^-_1 \rightarrow 0^+_1)$ $e^2b^2$</th>
<th>$E2SD$ (e b)</th>
<th>$E2DD$ (e b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{180}$W</td>
<td>0.850</td>
<td>0.098957</td>
<td>-0.292717</td>
</tr>
<tr>
<td>$^{182}$W</td>
<td>0.840</td>
<td>0.105548</td>
<td>-0.312213</td>
</tr>
<tr>
<td>$^{184}$W</td>
<td>0.756</td>
<td>0.108012</td>
<td>-0.319503</td>
</tr>
<tr>
<td>$^{186}$W</td>
<td>0.700</td>
<td>0.112815</td>
<td>-0.333711</td>
</tr>
<tr>
<td>$^{188}$W</td>
<td>0.600</td>
<td>0.114208</td>
<td>-0.337830</td>
</tr>
<tr>
<td>$^{190}$W</td>
<td>0.414</td>
<td>0.104653</td>
<td>-0.309566</td>
</tr>
</tbody>
</table>

A comparison between the experimental [15-20] and calculated $B(E2; 2^-_1 \rightarrow 0^+_1)$ are shown Fig. 7: the results are quite well for all isotopes under study.

The quadrupole moment ($Q$) is an important property for nuclei and is defined as the deviation from the spherical charge distribution inside the nucleus and from the quadrupole moment we can determine if the nucleus is spherical, deformed oblate or prolate shapes.

Fig. 7. The experimental and theoretical $B(E2; 2^-_1 \rightarrow 0^+_1)$ for $^{180-190}$W.
Figure 8 shows the comparison between the experimental quadrupole moments [16-20] with the present calculation for the $2_1^+$ level.

4. Conclusions

In the present work we studied systematically the lower and higher states of lower and higher bands, the B(E2) transition probabilities, and electric quadrupole moment $Q_{2_1^+}$ for the even-even $^{180-190}$W isotopes which lies in the transitional region SU(3)-O(6) of IBM-1. Good global agreement were obtained in comparison with the most recent experimental data and the model with the best fitted parameters proves that the nuclei $^{188}$W and $^{190}$W have high deformation and tend to be near O(6) limit than to SU(3) limit.

REFERENCES